history of man's best friend.

The findings should aid in tracking down disease genes, says Ostrander. She can now expand the search for a gene in one breed to other breeds shown to be related by their microsatellite compositions. Having a larger sample will make it easier to detect the mutation at fault. "This is what I see as the most powerful use of the data," she notes.

The dog offers other advantages over humans for gene hunts, says Sutter. To find the mutated genes underlying complex diseases such as cancer, geneticists look for base changes along the DNA where the implicated gene seems to be. Initial analyses suggest that geneticists will need to gather about 400,000 base differences—called single nucleotide polymorphisms—in the human genome to begin to pin down a problematic gene implicated in a disease.

But as Sutter reported at the Cold Spring Harbor meeting, such gene tracking should be much easier in dogs. By incorporating genomic information from 20 dogs from each of five breeds and the previously published poodle sequence (*Science*, 26 September 2003, p. 1898), he calculated that the job can be accomplished with just 30,000 SNPs.

At the same meeting, Lindblad-Toh described her progress sequencing the genome of a boxer named Tasha, chosen because the breed has very little genetic variation. Working with Ostrander and more than two dozen collaborators, Lindblad-Toh has sequenced enough DNA to cover the genome more than seven times over and expects that the consortium will put these data together into a high-quality draft. Once that goes public, which should occur in the next few weeks, finding disease genes in dogs will be even easier.

Dog breeders should be proud.

–ELIZABETH PENNISI

NUMBER THEORY

Proof Promises Progress in Prime Progressions

The theorem that Ben Green and Terence Tao set out to prove would have been impressive enough. Instead, the two mathematicians wound up with a stunning breakthrough in the theory of prime numbers. At least that's the preliminary assessment of experts who are looking at their complicated 50-page proof.

Green, who is currently at the Pacific Institute for the Mathematical Sciences in Vancouver, British Columbia, and Tao of the University of California (UC), Los Angeles, began working 2 years ago on the problem of arithmetic progressions of primes: sequences of primes (numbers divisible only by themselves and 1) that differ by a constant amount. One such sequence is 13, 43, 73, and 103, which differ by 30.

In 1939, Dutch mathematician Johannes van der Corput proved that there are an infinite number of arithmetic progressions of primes with three terms, such as 3, 5, 7 or 31, 37, 43. Green and Tao hoped to prove the same result for four-term progressions. The theorem they got, though, proved the result for prime progressions of *all* lengths.

"It's a very, very spectacular achievement," says Green's former adviser, Timothy Gowers of the University of Cambridge, who received the 1998 Fields Medal, the mathematics equivalent of the Nobel Prize, for work on related problems. Ronald Graham, a combinatorialist at UC San Diego, agrees. "It's just amazing," he says. "It's such a big jump from what came before."

Green and Tao started with a 1975 theorem by Endre Szemerédi of the Hungarian Academy of Sciences. Szemerédi proved that arithmetic progressions of all lengths crop up in any positive fraction of the integers—basically, any subset of integers whose ratio to the whole set doesn't dwindle away to zero as the numbers get larger and larger. The primes don't qualify, because they thin out too rapidly with increasing size. So Green and Tao set out to show that Szemerédi's theorem still holds when the integers are replaced with a smaller set of numbers with special properties, and then to prove that the primes constitute a positive fraction of that set.

To build their set, they applied a branch of mathematics known as ergodic theory (loosely speaking, a theory of mixing or averaging) to mathematical objects called pseudorandom numbers. Pseudorandom numbers are not truly random, because they are generated by rules, but they behave as random numbers do for certain mathematical purposes. Using these tools, Green and Tao constructed a pseudorandom set of primes and "almost primes," numbers with relatively few prime factors compared to their size.

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83
89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167
173 179 181 191 193 197 **199** 211 223 227 229 233 239 241 251
257 263 269 271 277 281 283 293 307 311 313 317 331 337 347
349 353 359 367 373 379 383 389 397 401 **409** 419 421 431 433
439 443 449 457 461 463 467 479 487 491 499 503 509 521 523
541 547 557 563 569 571 577 587 593 599 601 607 613 617 **619**
631 641 643 647 653 659 661 673 677 683 691 701 709 719 727
733 739 743 751 757 761 769 773 787 797 809 811 821 823 827
829 839 853 857 859 863 877 881 883 887 907 911 919 929 937
941 947 953 967 971 977 983 991 997 1009 1013 1019 1021 1031
1033 **1039** 1049 1051 1061 1063 1069 1087 1091 1093 1097 1103
1109 1117 1123 1129 1151 1153 1163 1171 1181 1187 1193 1201
1213 1217 1223 1229 1231 1237 **1249** 1259 1277 1279 1283 1289
1291 1297 1301 1303 1307 1319 1321 1327 1361 1367 1373 1381
1399 1409 1423 1427 1429 1433 1439 1447 1451 1453 **1459** 1471
1481 1483 1487 1489 1493 1499 1511 1523 1531 1543 1549 1553
1559 1567 1571 1579 1583 1597 1601 1607 1609 1613 1619 1621
1627 1637 1657 1663 1667 **1669** 1693 1697 1699 1709 1721 1723
1733 1741 1747 1753 1759 1777 1783 1787 1789 1801 1811 1823
1831 1847 1861 1867 1871 1873 1877 **1879** 1889 1901 1907 1913
1931 1933 1949 1951 1973 1979 1987 1993 1997 1999 2003 2011
2017 2027 2029 2039 2053 2063 2069 2081 2083 2087 **2089** 2099

Prime suspect. Arithmetic progressions such as this 10-prime sequence are infinitely abundant, if a new proof holds up.

The last step, establishing the primes as a positive fraction of their pseudorandom set, proved elusive. Then Andrew Granville, a number theorist at the University of Montreal, pointed Green to some results by Dan Goldston of San Jose State University in California and Cem Yildirim of Boğaziçi University in Istanbul, Turkey.

Goldston and Yildirim had developed techniques for studying the size of gaps between primes, work that culminated last year in a dramatic breakthrough in the subject—or so they thought. Closer inspection, by Granville among others, undercut their main result (*Science*, 4 April 2003, p. 32; 16 May 2003, p. 1066), although Goldston and Yildirim have since salvaged a less far-ranging finding. But some of the mathematical machinery that these two had set up proved to be tailor-made for Green and Tao's research. "They had actually proven exactly what we needed," Tao says.

The paper, which has been submitted to the *Annals of Mathematics*, is many months from acceptance. "The problem with a quick assessment of it is that it straddles two areas," Granville says. "All of the number theorists who've looked at it feel that the number-theory half is pretty simple and the ergodic theory is daunting, and the ergodic theorists who've looked at it have thought that the ergodic theory is pretty simple and the number theory is daunting."

Even if a mistake does show up, Granville says, "they've certainly succeeded in bringing in new ideas of real import into the subject." And if the proof holds up? "This could be a turning point for analytic number theory," he says.

–BARRY CIPRA