P. A. M. Dirac
and the Beauty of Physics

He preferred the beautiful theory to the fact-buttressed ugly one because, as he noted, facts change. He proved his point by predicting the existence of antimatter

by R. Corby Hovis and Helge Kragh

At the University of Moscow, distinguished visiting physicists are asked to leave on a blackboard some statement for posterity. Niels Bohr, father of the quantum theory of the atom, inscribed the motto of his famous principle of complementarity, "Contraria non contradictoria sed complementa sunt" (opposites are not contradictory but complementary). Hideki Yukawa, pioneer of the modern theory of the strong nuclear force, chalked up the phrase "In essence, nature is simple." Paul Adrien Maurice Dirac chose the epigraph "A physical law must possess mathematical beauty."

Exactly 30 years ago Dirac wrote in these pages, "God is a mathematician of a very high order, and He used very advanced mathematics in constructing the universe" [see "The Evolution of the Physicist's Picture of Nature," SCIENTIFIC AMERICAN, May 1963]. Inspired by the views of Albert Einstein and Hermann Weyl, Dirac, more than any other modern physicist, became preoccupied with the concept of "mathematical beau-

ty" as an intrinsic feature of nature and as a methodological guide for its scientific investigation. "A theory with mathematical beauty is more likely to be correct than an ugly one that fits some experimental data," he asserted.

Dirac's focus on the aesthetics and logic of mathematical physics, coupled with his legendary reticence and introversion, made him an enigmatic figure among the great 20th-century scientists [*see box on pages 106 and 107*]. Sadly, his extreme rationalism also led him into sterile byways after some amazingly successful early years. Between the ages of 23 and 31, Dirac unveiled an original and powerful formulation of quantum mechanics, a quantum theory of the emission and absorption of radiation by atoms (a primitive but important version of quantum electrodynamics), the relativistic wave equation of the electron, the idea of antiparticles and a theory of magnetic monopoles. Yet few of his subsequent contributions had lasting value, and none had the revolutionary character of his earlier work.

Dirac was born in 1902 in Bristol, England, the second of three children in a family that today would be branded as dysfunctional. The bane of the family was its head, Charles Adrien Ladislas Dirac, who had emigrated from Switzerland to England around 1890 and then met and married Florence Hannah Holten, the daughter of a ship's captain. Charles made a living teaching his native language, French, at the Merchant Venturers' Technical College in Bristol, where he was infamous as a rigid disciplinarian. He ran the Dirac household according to the same principles of regimental decorum. By avoiding displays of feeling and equating parental love with discipline, he imprisoned his children in a domestic tyranny that isolated them from so-

cial and cultural life. Unable or unwilling to revolt, Paul sank into the safety of silence and distanced himself from his father. These unhappy years scarred him for life. When Charles Dirac died in 1936, Paul did not grieve. "I feel much freer now," he wrote to his wife.

Fortunately, Paul had a rich interior world to which he could retreat. Early in life he showed an aptitude for mathematics. At age 12, he enrolled in the Merchant Venturers' Technical College. This school, unlike most others at the time, offered not a classical education in Latin and Greek but a modern curriculum in science, modern languages and the practical arts. These studies suited Dirac well, for as he said, he "did not appreciate the value of old cultures." After completing this secondary school program, he entered another institution housed in the same buildings, the Engineering College of the University of Bristol. There he prepared for the career of electrical engineer, not out of real fervor for the work but because he thought it would please his father.

The engineering curriculum gave short shrift to subjects outside applied

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HE WAS TALL, gaunt, awkward, and extremely taciturn," wrote the German physicist and biologist Walter Elsasser. "He had succeeded in throwing everything he had into one dominant interest. He was a man, then, of towering magnitude in one field, but with little interest and competence left for other human activities.... In other words, he was the prototype of the superior mathematical mind; but while in others this had coexisted with a multitude of interests, in Dirac's case everything went into the performance of his great historical mission, the establishment of the new science, quantum mechanics, to which he probably contributed as much as any other man."

physics and mathematics. Despite these omissions, Dirac became fascinated by and soon mastered Einstein's new theories of space, time and gravity—the special and general theories of relativity.

When Dirac graduated with first-class honors in 1921, the postwar economic depression seemed likely to leave him without a job. He was rescued by a scholarship to study mathematics at Bristol, after which he proceeded, in the fall of 1923, to graduate study in applied mathematics and theoretical physics at the University of Cambridge.

Cambridge was then home to such established scientists as Joseph Larmor, J. J. Thomson, Ernest Rutherford, Arthur Stanley Eddington and James Jeans, as well as to such rising stars as James Chadwick, Patrick Blackett, Ralph Fowler, Edward A. Milne, Douglas R. Hartree
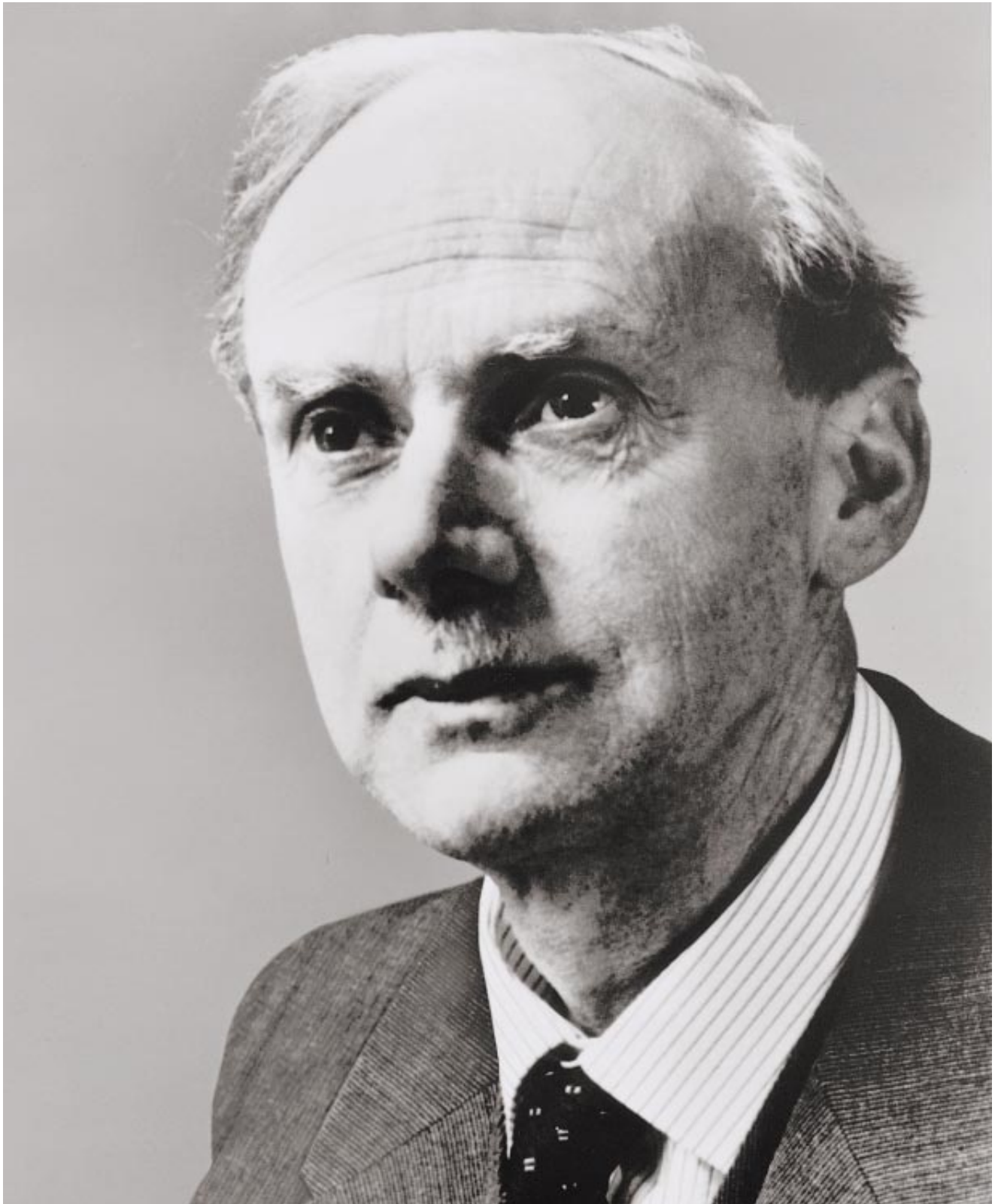
Memorabilia

In 1931, when he was a lecturer and fellow at Cambridge, **Nevill Mott** wrote to his parents: "Dirac is rather like one's idea of Gandhi. We had him to supper here.... It was quite a nice little supper but I am sure he would not have minded if we had only given him porridge. He goes to Copenhagen by the North Sea route because he thinks he ought to cure himself of being sea sick. He is quite incapable of pretending to think anything that he did not really think. In the age of Galileo he would have been a very contented martyr."

J. Robert Oppenheimer

Dirac once attended a luncheon with **Eugene Wigner** and **Michael Polanyi**. There was a lively discussion about science and society, during which Dirac did not say a word. Asked to speak up and give his opinion, he responded, "There are always more people willing to speak, than willing to listen."

A French physicist, who spoke English with great difficulty, once called on Dirac. Dirac listened patiently as the fellow tried to find the right English words to get his point across. Dirac's sister then came into the room and asked Dirac something in French, to which he also replied in fluent French. Naturally, the visitor was indignant and burst out, "Why did you not tell me that you could speak French?" Dirac's terse answer: "You never asked me."

When Dirac passed through Berkeley on his way to Japan in 1934, **J. Robert Oppenheimer** met him and offered him two books to read during the voyage. Dirac politely refused, saying that reading books interferes with thought. Once the Russian physicist **Peter Kapitza** gave Dirac an English translation of Fëdor Dostoevski's *Crime and Punishment*. After some time had passed, Kapitza asked Dirac if he had enjoyed the book. His only comment was: "It is nice, but in one of the chapters the author made a mistake. He describes the Sun as rising twice on the same day." On advice, Dirac also read Leo Tolstoi's *War and Peace;* it took him two years.

Eugene Wigner

Peter Kapitza

and Peter Kapitza. Dirac was assigned Fowler as his supervisor, and from him Dirac learned atomic theory and statistical mechanics, subjects he had not previously studied. Of these years, he later recalled: "I confined myself entirely to the scientific work, and continued at it pretty well day after day, except on Sundays when I relaxed and, if the weather was fine, I took a long solitary walk out in the country."

Six months after arriving at the university, he published his first scientific paper; in the next two years he published 10 more. By the time he completed his Ph.D. dissertation in May 1926, he had discovered an original formulation of quantum mechanics and taught the first quantum mechanics course ever offered at a British university. Only 10 years after entering Cambridge, he would receive the Nobel Prize in Physics for his "discovery of new fertile forms of the theory of atoms…and for its applications."

The eight great years in Dirac's life began one day in August 1925, when he received from Fowler the proofs of a forthcoming article by Werner Heisenberg, a young German theorist [see "Heisenberg, Uncertainty and the Quantum Revolution," by David C. Cassidy; SCIENTIFIC AMERICAN, May 1992]. The article laid out the mathematical foundations of a revolutionary theory of atomic phenomena that would soon be known as quantum mechanics. Dirac immediately realized that Heisenberg's work opened up an entirely new way of looking at the world on an ultramicroscopic scale. During the next year, he reformulated Heisenberg's basic insight into an original theory of quantum mechanics that became known as q-number algebra, after Dirac's term for an "observable" physical quantity, such as position, momentum or energy.

Although Dirac's work quickly earned him widespread recognition, many of his results were derived contemporaneously by a strong group of theorists working in Germany, including Heisenberg, Max Born, Wolfgang Pauli and Pascual Jordan. Dirac openly competed with them.

Born, Heisenberg and Jordan elaborated Heisenberg's initial scheme in terms of the mathematics of matrices. Then, in the spring of 1926, the Austrian physicist Erwin Schrödinger produced another quantum theory, wave mechanics, which led to the same results as the more abstract theories of Heisenberg and Dirac and lent itself more readily to computation. Many physicists suspected that the three systems were merely special representations of a more general theory of quantum mechanics.

During a six-month stay at the Institute for Theoretical Physics in Copenhagen, Dirac found the general theory for which so many researchers had hoped—a framework that subsumed all the special schemes and provided definite rules for transforming one scheme into another. Dirac's "transformation theory," together with a similar theory worked out at the same time by Jordan, provided the foundation for all later developments in quantum mechanics.

On December 26, 1927, the English physicist Charles G. Darwin (grandson of the famous naturalist) wrote to Bohr: "I was at Cambridge a few days ago and saw Dirac. He has now got a completely new system of equations for the electron which does the spin right in all cases and seems to be 'the thing.' His equations are first order, not second, differential equations!"

Dirac's equation for the electron was indeed "the thing," for it at once satisfied the requirements of the special theory of relativity and accounted for

Dirac shunned publicity. At first he was inclined not to accept his Nobel Prize. On the day his appointment to the Lucasian chair was announced, he escaped to the zoo to avoid the many congratulations. He refused all honorary degrees—although many were awarded him in his absence and apparently without his acquiescence.

Around 1950 Dirac was assigned to supervise **Dennis Sciama's** graduate studies at Cambridge. One day Sciama enthusiastically entered Dirac's office, saying, "Professor Dirac, I've just thought of a way of relating the formation of stars to cosmological questions. Shall I tell you about it?" Dirac's reply: "No." End of conversation. Dirac did not seem to realize that his brevity and candor could be perceived as impoliteness or impudence.

When Dirac delivered lectures, he strove to present his text with a maximum of lucidity and directness. He considered it illogical to change his carefully chosen phrases just because they had not been understood. More than once, somebody in the audience asked him to repeat a point that had not been understood, meaning that the listener would like a further exposition. In such cases, Dirac would repeat exactly what he had said before, using the very same words.

In 1977 Dirac wrote: "Of all the physicists that I met, I think **Schrödinger** was the one that I felt to be most closely similar to myself. I found myself getting into agreement with Schrödinger more readily than with anyone else. I believe the reason for this is that Schrödinger and I both had a very strong appreciation of mathematical beauty. . . . It was a sort of act of faith with us that any equations which describe fundamental laws of Nature must have great mathematical beauty in them."
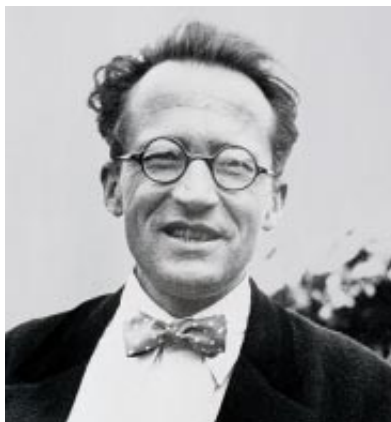
Erwin Schrödinger

the experimentally observed "spin" of the electron, which can take either of two values, $+\frac{1}{2}$ or $-\frac{1}{2}$, "up" or "down." Schrödinger's original equation had failed to do this because it was not relativistic, and its relativistic extension, the Klein-Gordon equation, could not account for the spin.

The use of only first derivatives, so impressive to Darwin, was crucial for two reasons. First, Dirac wanted to retain the formal structure of Schrödinger's equation, which contained a first derivative in time. Second, he needed to meet the strictures of relativity, which put space and time on an equal footing. Dirac's difficult reconciliation of the two criteria was at once beautiful and functional: when he applied the new equation to the case of an electron moving in an electromagnetic field, the correct value of the electron's spin came out automatically.

This deduction of a physical property from first principles impressed physicists, who referred to the equation as "a miracle" and "an absolute wonder" and set about to analyze its subtleties. This line of research eventually led to the birth of spinor analysis—a powerful mathematical tool for analyzing problems in virtually all branches of phys-

ics—and to the development of relativistic wave equations for particles having spin other than one-half. In another success, when Dirac and others applied his equation to the hydrogen atom, they were able to reproduce exactly the lines observed in its spectrum. Less than a year after publication, the Dirac equation had become what it remains: a cornerstone of modern physics.

A worshiper of mathematical logic, Dirac was also a master of intuition. These seemingly contradictory intellectual traits were nowhere exhibited more prominently than in his development of his theory of "holes" between 1929 and 1931. This theory illuminated an entire world that had escaped the notice of physicists.

The theory arose from Dirac's realization that his equation pertained not only to familiar, positive-energy electrons but also to electrons having *negative* energy. Such particles would exhibit quite peculiar properties. Furthermore, positive-energy particles would routinely drop down into these negative-energy states, bringing the collapse of the world around us!

In late 1929 Dirac found a way out of the conundrum created by the appar-

ent necessity of negative-energy electrons in nature. He imagined the vacuum to constitute a uniform "sea" of negative-energy states all *filled* by electrons. Since the Pauli exclusion principle prohibits two electrons from occupying the same quantum state, positive-energy electrons would be kept above the invisible sea, to form the "excited" states observed in nature. An excited state could also be created by pouring in enough positive energy to raise an electron from the sea, a process that would leave a "hole" into which another negative-energy electron could fall. "These holes will be things of positive energy and will therefore be in this respect like ordinary particles," Dirac wrote.

But with what particle could a hole be identified? At the time, there were two plausible candidates, both of which Dirac considered: the proton and the positive electron. His first choice, the proton, faced two major difficulties almost immediately. First, one would expect an electron occasionally to jump down and fill a hole, in which case the two particles would annihilate in a flash of light (gamma rays). Such proton-electron annihilations had never been observed. Second, it became apparent that the correct candidate needed to be identical to the electron in all respects except for electric charge—yet the proton was known to be nearly 2,000 times more massive than the electron.

Nevertheless, Dirac, prompted by a desire for simplicity, at first favored the proton as the hole. In 1930 the electron and the proton were the only known fundamental particles, and he did not relish introducing a new and unobserved entity. Moreover, if protons could be interpreted as negative-energy states vacated by electrons, the number of elementary particles would collapse to one, the electron. Such a simplification would be "the dream of philosophers," Dirac declared.

But the objections to his initial interpretation of holes soon became overpowering, and in May 1931 he settled, reluctantly, on the second candidate for the hole, the antielectron, "a new kind of particle, unknown to experimental physics, having the same mass and opposite charge to an electron." The complete symmetry between positive and negative charges in his theory further impelled him to admit the antiproton to the realm of theoretical existence. Thus did Dirac double the number of respectable elementary particles and set the stage for speculations about entire worlds made of antimatter. He also argued for the existence of another hypothetical particle, the magnetic monopole, which would carry an isolated

magnetic charge analogous to the electron's or proton's electric charge. Even today there is no conclusive experimental evidence for monopoles [see "Superheavy Magnetic Monopoles," by Richard A. Carrigan, Jr., and W. Peter Trower; Scientific American, April 1982].

In September 1932 Dirac was elected to the Lucasian Chair of Mathematics at Cambridge, a professorship that Isaac Newton had once held for 30 years and that Dirac would keep for 37 (it is now occupied by Stephen W. Hawking). That same month, a young experimenter at the California Institute of Technology, Carl D. Anderson, submitted a paper to the journal *Science* that described his apparent detection, in cosmic rays, of "a positively charged particle having a mass comparable with that of an electron." Although this discovery was not at all inspired by Dirac's theory, the new particle, dubbed the "positron," became generally equated with Dirac's antielectron. When he accepted his Nobel Prize in Stockholm in December 1933, the 31-year-old Dirac lectured on the "Theory of Electrons and Positrons." Three years later Anderson, also at age 31, received the Nobel Prize for raising Dirac's particle from the realm of the hypothetical.

Quantum electrodynamics (QED) is the name given to a quantum theory of the electromagnetic field. By the mid-1930s, attempts to formulate a satisfactory relativistic quantum field theory had reached a state of crisis, and many physicists concluded that a drastic change in fundamental physical ideas was needed. Dirac had made pathbreaking contributions to QED in the late 1920s and was painfully aware of the formal shortcomings of the existing theoretical framework, which was built mainly around a theory advanced by Heisenberg and Pauli in 1929. Dirac called the theory illogical and "ugly." Moreover, calculations using it led to divergent integrals—infinities—to which no physical meaning could be attached. In 1936 Dirac worked out an alternative theory in which energy was not conserved. Although this radical proposal was quickly refuted by experiments, Dirac continued to criticize the Heisenberg-Pauli theory and to search—almost obsessively—for a better one. Looking back on his career in 1979, he wrote, "I really spent my life mainly trying to find better equations for quantum electrodynamics, and so far without success, but I continue to work on it."

One logical route toward a better QED would be to use, as a springboard, an improved classical theory of the electron. In 1938 Dirac followed this strategy and produced a classical-relativistic theory of the electron that greatly improved on the old theory that H. A. Lorentz had framed near the beginning of the century. Dirac's theory resulted in an exact equation of motion for an electron treated as a point particle. Because the theory avoided infinities and other ill-defined terms, it seemed likely to lead to a divergence-free QED. But creating a satisfactory quantum mechanical version of the theory turned out to be more troublesome than Dirac had anticipated. He fought with this problem for more than 20 years— in vain.

During 1947 and 1948, a new theory of QED emerged that resolved, in a practical sense, the difficulty of the infinities that had previously ruined calculations. The pioneers of the new theory, Sin-itiro Tomonaga in Japan and Richard Feynman, Julian Schwinger and Freeman Dyson in the U.S., proposed a procedure called "renormalization," in which the infinite quantities in theoretical calculations were effectively replaced by the experimentally measured values for the mass and charge of the electron. This procedure of (in effect) subtracting infinities made possible extremely accurate predictions, and the theory's many empirical successes convinced physicists to adopt renormalization as *the* method for doing QED.

Dirac, however, resisted the renormalization approach, judging it to be as "complicated and ugly" as the older one of Heisenberg and Pauli. A theory that operates with ad hoc mathematical tricks not directly dictated by basic physical principles cannot be good, he argued, no matter how well it matches experimental results. But his objections were mostly ignored. At the end of his life, he was forced to admit not only that he had become isolated in the physics community but also that none of his many proposals to reconstruct QED had succeeded.

Dirac's fight for an alternative quantum field theory did have some significant by-products, however. One of these was his important classical theory of the electron, mentioned earlier. Another was a new notation for quantum mechanics, known as the "bra-ket," or "bracket," formalism, which elegantly introduced into the subject the powerful mathematics of vector spaces (or "Hilbert spaces," as they are sometimes called). This formalism became widely known through the third (1947) edition of his influential textbook *The Principles of Quantum Mechanics* and has been the preferred mathematical language for the subject ever since.

In general, Dirac worked only in rather specialized areas of quantum theory. So it was somewhat surprising when, in 1937, he ventured into cosmology with a new idea and then developed it into a definite model of the universe. His interest was largely inspired by two of his former teachers at Cambridge, Milne and Eddington, and by discussions with the talented young Indian astrophysicist Subrahmanyan Chandrasekhar, whose graduate work at Cambridge Dirac partly supervised. In the early 1930s Eddington had embarked on an ambitious, unorthodox research program, aiming to deduce the values of the fundamental constants of

It seems reasonable to assume that not all the states of negative energy are occupied, but that there which can be described by a wave function, like an X-ray are a few vacancies or "holes". Such a hole would appear experimentally as a thing with +ve energy, since to make the hole disappear (i.e. to fill it up) one would have to put −ve energy into it. Further, one can easily

CONCEPT OF ANTIMATTER, which Dirac introduced in 1931, grew directly from his theory of "holes," outlined here in a letter to Niels Bohr dated November 26, 1929. It illustrates Dirac's characteristic clarity, conciseness and neat handwriting.

nature by bridging quantum theory and cosmology. This quest for a truly "fundamental theory," as Eddington called it, stretched rational inquiry into the realm of metaphysical speculation—producing, one critic charged, a "combination of paralysis of the reason with intoxication of the fancy." Dirac was skeptical of Eddington's imaginative claims but impressed by his philosophy of science, which emphasized the power of pure mathematical reasoning, and by his idea of a fundamental connection between the microworld and the macroworld.

In his first article on cosmology, Dirac focused on the very big "pure," or dimensionless, numbers that can be constructed by algebraically combining fundamental constants (such as the gravitational constant, Planck's constant, the speed of light and the charge and masses of the electron and proton) so that their units of measurement cancel in division. He argued that only these large numbers have profound significance in nature.

For example, it was known that the ratio of the electric force between a proton and electron to the gravitational force between the same two particles is a very large number, about 10^{39}. Curiously, Dirac noted, this number approximates the age of the universe (as then estimated) when that age is expressed in terms of an appropriate unit of time, such as the time needed for light to cross the diameter of a classical electron.

Dirac knew of several such correlations between large pure numbers, but instead of considering them to be mere coincidences, he held that they formed the essence of an important new cosmological principle, which he christened the Large Number Hypothesis: "Any two of the very large dimensionless numbers occurring in Nature are connected by a simple mathematical relation, in which the coefficients are of the order of magnitude unity."

From this principle, Dirac readily—and controversially—concluded that the gravitational "constant" G is inversely proportional to the age of the universe and hence must be steadily decreasing with cosmic time.

By 1938 Dirac had derived several empirically testable consequences from the Large Number Hypothesis and had outlined his own model of the universe based on that principle. But most physicists and astronomers—who had become increasingly annoyed by the rationalistic approach to cosmology—dismissed his ideas. Only decades later, in the 1970s, did Dirac resume work in cosmology, mostly on the basis of his original theory. He defended the Large

Predicting Hydrogen's Alpha Line

Hydrogen spectrum's alpha line illustrates the advances in atomic theory since Niels Bohr first explained it in 1913 as the result of a single quantum transition. When improved experiments revealed a fine structure in the line, Arnold Sommerfeld combined Bohr's atomic theory with Einstein's special theory of relativity to explain the components as the result of several transitions. Attempts to derive Sommerfeld's result from the new quantum mechanics failed until 1928, when Dirac's theory of the electron proved to reproduce exactly Sommerfeld's old equation. Later measurements revealed still finer structure, which was explained theoretically in the late 1940s by the modern quantum electrodynamics of Julian Schwinger, Richard Feynman and Sin-itiro Tomonaga. Dirac disliked this new theory because it was, he said, "just a set of working rules," not a complete theory built on a "sound and beautiful" foundation.

BOHR (1913)   DIRAC (1928)   SCHWINGER-FEYNMAN-TOMONAGA (1947–48)

INTENSITY

FREQUENCY
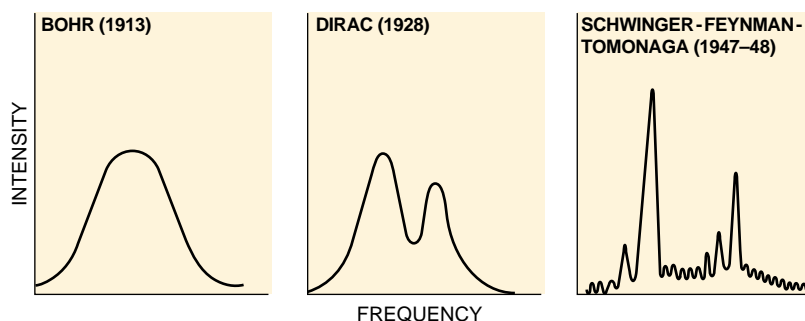
Number Hypothesis and his prediction of a varying gravitational constant against observationally based objections and attempted to modify his model to accommodate new discoveries such as the cosmic microwave background radiation. His efforts failed to gain recognition, and he remained—in cosmology as in QED—a figure estranged from the mainstream of research.

Dirac was wedded to his work, and his colleagues had long considered him an inveterate bachelor. It therefore came as a surprise when in 1937 he married Margit Wigner, sister of prominent Hungarian physicist Eugene Wigner. Margit was a widow; she brought a son and a daughter from her previous marriage, and with Paul he had two girls. Not surprisingly, he remained detached from family life. "It is the irony which only life can produce that Paul suffered severely from his father, who had the same difficulties with his family," Margit has written. "Paul, although not a domineering father, kept himself too aloof from his children. That history repeats itself is only too true in the Dirac family."

Dirac never developed an interest in art, music or literature, and he seldom went to the theater. The only hobbies to which he devoted much time were hiking in the mountains and traveling. He was a tireless walker, and on tours

he often demonstrated stamina that amazed those who knew him only from conferences or dinner parties. His travels took him around the world three times, and he climbed some of the highest peaks in Europe and America.

In September 1969 Dirac retired from the Lucasian chair. The next year, he and Margit decided to leave England permanently for the warm climate of Florida, where he accepted a faculty position at Florida State University in Tallahassee. He remained productive and participated in many conferences until his health began to fail. He died in Tallahassee in October 1984.

FURTHER READING

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