by Ian Stewart

What a Coincidence!

Some years ago a friend of mine was on his honeymoon, camping in a remote part of Ireland. He and his wife were walking along a deserted beach when in the distance they saw two people coming toward them. The other couple turned out to be my friend's boss and his new wife. Neither couple knew of the other's plans: it was a coincidence. Such a striking coincidence, in fact, that it always makes a good story.

People seem endlessly fascinated by coincidences, but should we be impressed by these fluky events, especially when they seem to happen to everyone? Robert Matthews, a British journalist and mathematician whose work has been noted in this column before ("The Anthropomurphic Principle," December 1995; "The Interrogator's Fallacy," September 1996), thinks not. In a recent issue of *Teaching Statistics* (Spring 1998), he and co-author Fiona Stones examine one of the most common types of coincidence: people who share the same birthday. Their conclusion is that we are too impressed by coincident birthdays because we have a very poor intuition of how likely such events are.

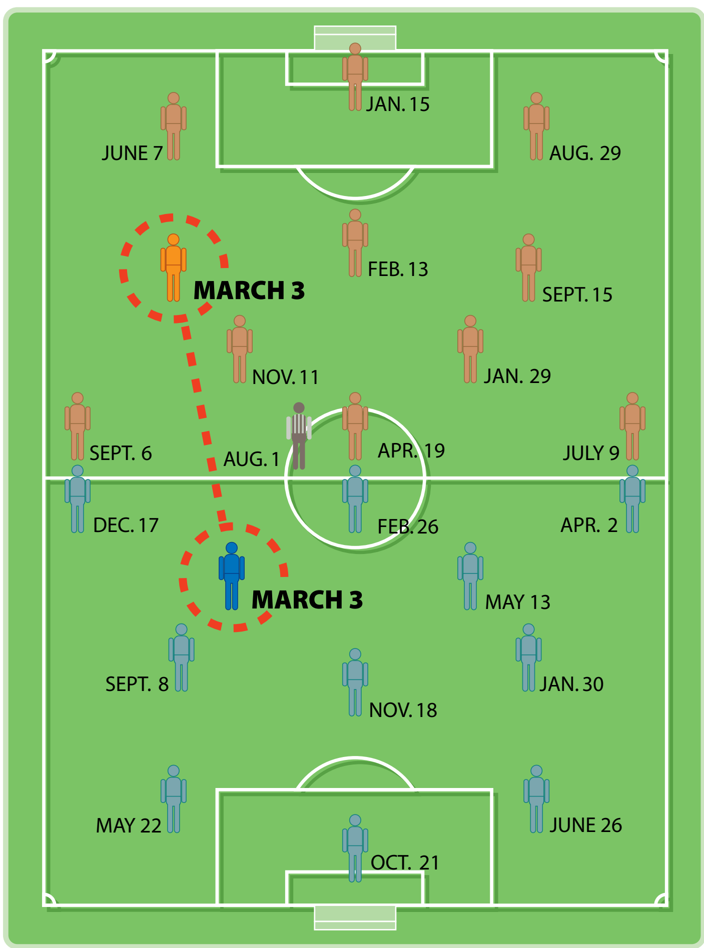
How many people must be in a room to make it more likely than not that at least two of them will have the same birthday? By "birthday" I mean day and month but not year. To keep things simple, I'll ignore February 29, so there are 365 different birthdays. I'll also assume that each birthday is equally likely to occur, which is not entirely true: more children are born at some times of the year than at others. Taking these extra factors into account would complicate the analysis without greatly changing the conclusions.

Okay, how many people are in the room? A hundred? Two hundred? When researchers posed this question to university students, the median of their estimates was 385. The popular guess is obviously too high, because as soon as the room contains 366 people (or 367 if we include February 29) a coincidence is guaranteed. In fact, the correct number is much lower: only 23 people.

When doing these kinds of calculations, it is often easier to determine the probability that an event does *not* happen. If we know this number, then all we have to do to get the probability that the event *does* happen is to subtract the number from 1. For instance, when does the event "at least two people share the same birthday" not happen? When all their birthdays are different. Suppose we start with just one person in the room and then bring in more people one at a time. We can calculate the probability that the new person has a different birthday from all the previous ones. As people enter the room, the probability that all their birthdays differ steadily decreases—and the probability of a coincidence increases. Now, an event that is more likely to happen than not has a probability greater than $1/2$. As soon as the probability of no differing birthdays drops below $1/2$, we know that a coincidence—at least two people sharing the same birthday—has become more likely than not.

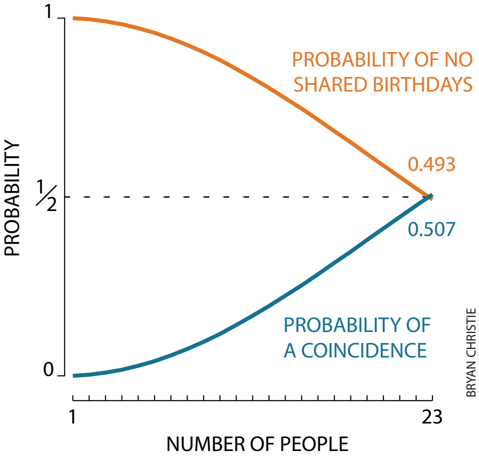
With only one person—let's call him Alfred—there is no possibility of a coincidence, so the probability of unique birthdays is certain, or 1. Now Betty enters the room. There are 365 possible birthdays, but Alfred has used up one of them. That leaves 364 possibilities for Betty if their birthdays are to be different. So the probability that their two birthdays differ is $364/365$. Next, Carla comes in. Now there are only 363 unused birthdays, so the probability that Carla has a birthday that differs from the other two is $363/365$. The combined probability that all three birthdays are different is $(364/365) \times (363/365)$.

We're starting to see a pattern. When Diogenes comes into the room, the probability of differing



BRYAN CHRISTIE

HYPOTHETICAL SOCCER MATCH includes two players who share the same birthday. Every game is likely to have at least one birthday coincidence among the 22 players and the referee.



BRYAN CHRISTIE

***BIRTHDAY COINCIDENCE** becomes more likely than not when the number of people rises to 23.*

birthdays is $(364/365) \times (363/365) \times (362/365)$. In general, after the nth person has entered the room, the probability that all n birthdays are different is $(364/365) \times (363/365) \times \ldots \times ((365 - n + 1)/365)$. All we have to do now is compute successive values of this expression and see when it drops below $1/2$. The graph above shows the results. With 22 people the probability of all birthdays being different is 0.524, but with 23 people it is 0.493. So when the 23rd person enters, the probability that at least two of the people present have the same birthday becomes $1 - 0.493 = 0.507$, or slightly more likely to happen than not.

Test the theory at parties of more than 23 people. Take bets. In the long run, you'll win. At big parties you'll win easily. Most partygoers will guess that a coincidence is unlikely because they focus on a misleading aspect of the problem: the number of people in the room. Although 23 is small, there are 253 different pairings among 23 people. (If n is the number of people, the number of pairings is $n \times (n-1)/2$.) That's a lot larger and a lot more relevant to the probability of a coincidence.

Matthews and Stones tested the prediction in another way. In a soccer match there are 23 people on the field: two teams of 11 players each, plus the referee. So the prediction is that among such a group, more often than not, two birthdays will coincide. Matthews and Stones looked at soccer matches in the U.K.'s Premier Division played on April 19, 1997. Out of 10 games, there were six with birthday coincidences, four without.

In fact, in two of the matches there were two coincidences! In the game pitting Liverpool against Manchester United, two players had birthdays on January 21, and two had birthdays on August 1. In the showdown between Chelsea and Leicester City, the two shared birthdays were November 1 and December 22. But this fluke is also predicted by probability theory—the chance of two shared birthdays among 23 people is 0.111, so it is likely to happen in one out of nine soccer matches. The chance of three shared birthdays is 0.018, and the chance of a triple coincidence—three people out of 23 sharing the same birthday—is 0.007, or likely to happen in one out of 143 games.

Now for a slightly different question: How many people, in addition to yourself, must be in a room to make it more likely than not that one of them will share *your* birthday? One might guess the answer is $(364/2) + 1$, or 183, because there are 364 birthdays that are different from yours, and if more than half that number of people are in the room, you're likely to share a birthday with someone. But the correct answer is 253.

To calculate the number, use the same technique as before: find the probability that the birthdays remain different from yours and then subtract from 1. Suppose that you're already in the room and that other people come in one by one—Alfred, Betty, Carla, Diogenes and so on. The probability that Alfred has a different birthday from yours is $364/365$. The probability that Betty has a different birthday is also $364/365$. And the same goes for Carla, Diogenes and everyone else. We're not interested in the coincidences shared by other people—say, Alfred and Carla both have birthdays on May 19. Doesn't matter: all that counts is whether a birthday is the same as yours. So after n people have entered the room, the probability that they all have different birthdays from yours is $(364/365)$ to the nth power. The first value of n for which this number is less than $1/2$ is $n = 253$; $(364/365)^{253} = 0.499$.

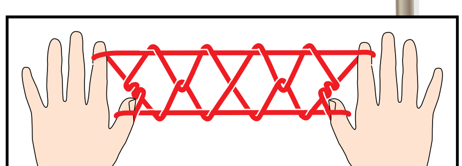
Incidentally, the fact that the answer to the second problem is the same as the number of pairings in the first problem (253 pairings for 23 people) seems not to have any mathematical significance. It seems to be a coincidence.

What do such calculations teach us? First, not to be unduly impressed by things that seem unlikely: maybe they're not. A pair of soccer players with the same birthday would probably be amazed by the coincidence, even though every soccer match is likely to have one. The players would probably remember the coincidence for years. But the 252 other pairs of players would not marvel at the fact that their birthdays did *not* coincide. Because we notice coincidences and ignore noncoincidences, we make the coincidences seem more significant than they really are. My friend's honeymoon encounter seems less striking if you think of how many other people he must have encountered during his life who were *not* his boss and spouse. SA

FEEDBACK

In the column "Cat's Cradle Calculus Challenge" (December 1997), I asked readers to devise a mathematical theory that would explain some aspects of string figures. Mark A. Sherman, editor of the *Bulletin of the International String Figure Association,* sent me several copies of his journal and its predecessor, the *Bulletin of String Figures Association,* containing articles that head in the right direction. Among them are an entire special issue devoted to the mathematical principles of string figures (*B.S.F.A.*, 1988), including a chapter on the beautiful pattern known as Indian diamonds (*below*), and an article on using string figures to teach math skills (*B.I.S.F.A.*, 1997).

This topic deserves a fuller exposition, and I hope to return to it in a future column. Meanwhile, readers may wish to contact the International String Figure Association at P. O. Box 5134, Pasadena, CA 91117, or at http://members.iquest.net/~webweavers/isfa.htm on the World Wide Web. *—I.S.*



DANA BURNS-PIZER